Waveform coding techniques.

Pulse code modulation.

Pulse code modulation is an analog to digital converter where the information contained in the instantaneous samples of an analog signal are represented by digital codes in a serial bit stream manner

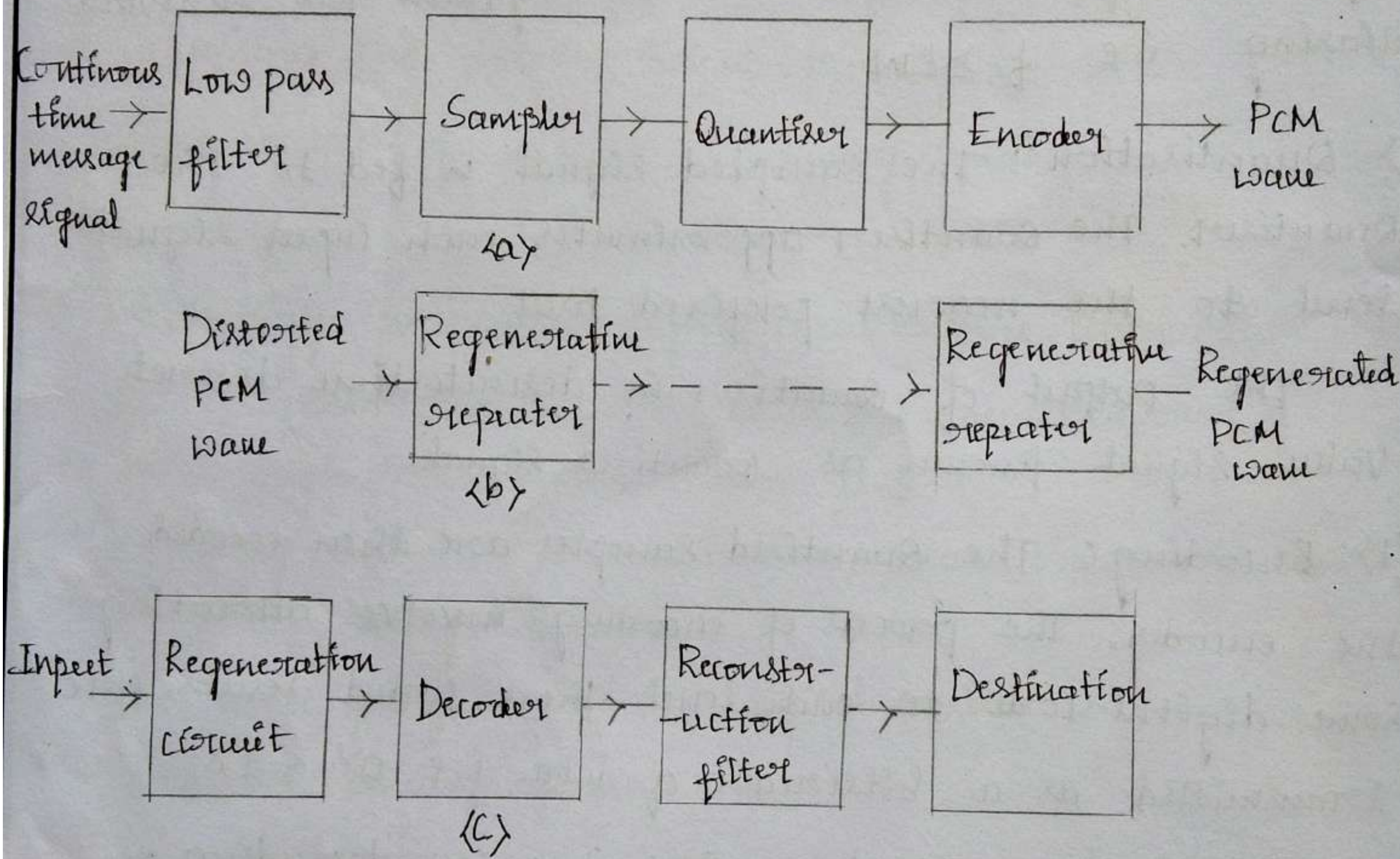


Fig. Basic elements of a PCM system

(a) Transmitter (b) Transmission path (c) Receiver

The Block diagram of PCM system consists of

a) Transmitter

b) Regenerative repeater

c) Receiver

1> The Low pass filter [pre-alias filter] is used before Sampler in order to limit the frequency greater than 'W' Hz. Hence message signal is bandlimited to W Hz.

2> Sampler : The Incoming message signal is sampled with a train of Narrow rectangular pulses. the sampling rate 'f_s' is selected above Nyquist rate to avoid aliasing i.e $f_s \geq 2W$

3> Quantisation : The sampled signal is fed to the Quantiser. The Quantiser approximates each input signal level to the nearest prefixed level.

The output of Quantiser is descrete time descrete value signal known as Quantized signal.

4> Encoding: The Quantized samples are then encoded in the encoder. The process of encoding involves allocating some digital code to each level. these coded levels are transmitted as a bitstream of data i.e 0's & 1's.

The Encoder output consists of pulses depending on the code combination.

5> Regenerative repeater; The PCM signal is reconstructed by means of regenerative repeater located at sufficiently closed spacing along the transmission path.

The regenerative networks are used at intermediate points between transmitter and receiver in order to boost up the pulse amplitude.

6> Decoder: The first operation in the receiver is to generate the received pulses.

The decoder converts binary coded signal to a approximated pulses of discrete magnitude.

7> Reconstruction filter: The final operation in the receiver is to recover the original analog signal. This is done by passing the decoder output through a LPF.

The output of LPF is an analog signal.

Advantages

1> Relatively inexpensive digital circuitry is involved in PCM

2> PCM signals can be multiplexed & transmitted over a common high speed communication link.

3> In long distance communication, clean waveforms can be regenerated using repeaters.

4> The noise performance of digital system is superior to that of an analog system.

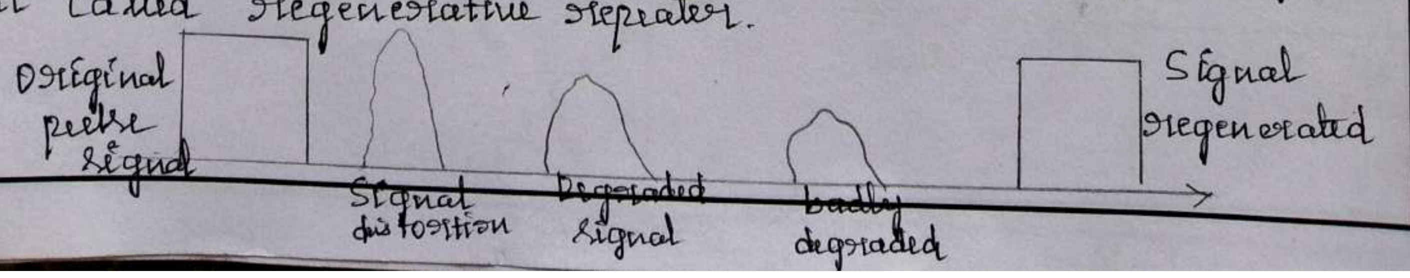
Pulse degradation

The shape of the pulse is affected mainly by two mechanisms

1> Unwanted electrical noise or other disturbance.

2> Nonideal transfer function of the transmission medium

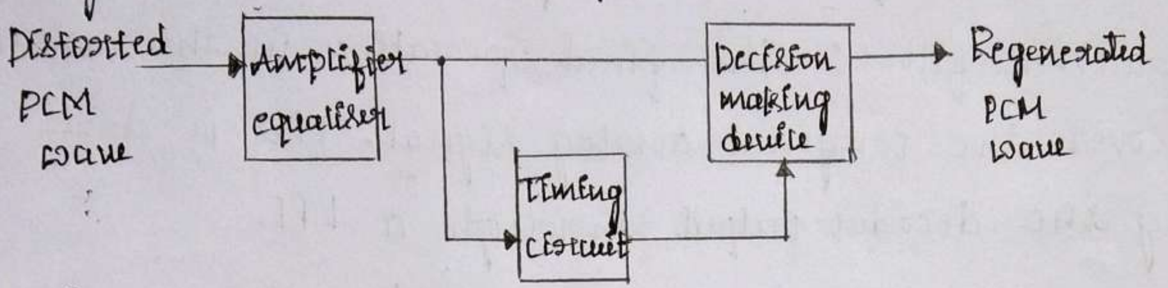
The shape of the pulse degraded as a function of the distance travelled. This degraded pulse can be regenerated by using circuit called regenerative repeater.



Regenerative repeater

The regenerative repeater is used to remove noise from the incoming PCM data.

Regenerative repeaters are installed at regular interval along the channel through which PCM signal is transmitted.



The regenerative repeater performs three basic operations

1) Equalization 2) Timing 3) Decision making

Equalizer: The equalizer shapes the received pulses so as to compensate for the effect of amplitude and phase distortion produced by the transmission channel.

2) Timing circuitry: The Timing circuit provides a periodic pulse train derived from the received pulses.

The output of amplifier is sampled at different sampling instants where signal to noise ratio is maximum.

3) Decision making device: The sample extracted is compared with a predetermined threshold [λ] in the decision making device. During each bit interval, a decision is made whether the received symbol is a '1' or '0'.

If the threshold is exceeded, a clean new pulse representing symbol '1' transmitted, otherwise a clean base line representing '0' is transmitted.

Quantisation

Quantization is a process of representing the sampled Values of the amplitude by a finite set of levels. It means Converting a continous-amplitude sample in to a discrete time signal.

* The Quality of a Quantizer output depends upon the number of quantization levels used.
* The discrete amplitudes of the quantized output are called as representation levels
* The spacing between two adjacent representation levels is known as step size.
* There are two types of Quantization.
  1) Uniform Quantization
  2) Non uniform Quantization.

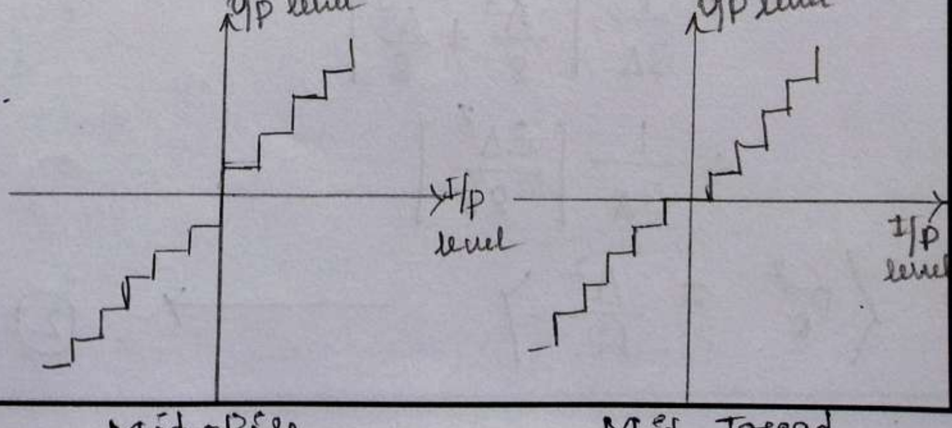
Quantization in which the quantization levels are uniformly spaced is termed as Uniform quantization.

Quantization in which quantization levels are unequal and relation between them is logarithmic is termed as Non-uniform Quantization

Uniform quantization.

There are two types of uniform quantization.

- Mid-Rise type
- Mid-Tread type.



Mid-Rise Mid-Tread

Quantisation noise and Signal to Noise ratio in PCM system

Derive the equation for Signal to quantization noise ratio if probability of overload is less than 10^{-4} in the case of a uniform quantizer. Further, if a binary code of n-bits ($n>6$), write the equation for (SNR).

>

Let the random variable 'Q' denotes the quantization error and 'q' its sample value.

Let us assume that the quantization error Q is uniformly distributed over a single quantizer interval 'Δ'.

Probability density function [PDF] of quantization error is

$$f_Q(q) = \begin{cases} 1/\Delta & \text{for } -\Delta/2 \le q \le \Delta/2 \\ 0 & \text{otherwise.} \end{cases}$$

The variance of quantization error is

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} (q-\mu)^2 f_Q(q)\, dq$$

Mean quantization error, $\mu = 0$

$$= \int_{-\Delta/2}^{\Delta/2} (q-0)^2\, 1/\Delta \cdot dq$$

$$= 1/\Delta \int_{-\Delta/2}^{\Delta/2} q^2 \cdot dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left[\frac{\Delta}{2}\right]^3 - \left[\frac{\Delta}{2}\right]^3\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right]$$

$$= \frac{1}{3\cancel{\Delta}}\left[\frac{2\Delta^{3}}{8_4}\right]$$

$$\left\langle \sigma_Q^2 = \frac{\Delta^2}{12} \right\rangle \longrightarrow (2)$$

The signal to quantization noise ratio [SNR] is given by

$$\frac{S}{N} = \frac{\text{Normalised Signal power}}{\text{Normalised Noise power.}} = \frac{\sigma_X^2}{\sigma_Q^2}$$

$$= \frac{\sigma_X^2}{\Delta^2/12}$$

$$(SNR) = \frac{12\,\sigma_X^2}{\Delta^2} \quad \text{———} \quad (3)$$

Let x_{max} denote the absolute value of the overload level of the quantizer & Δ denotes its step size.

The number of levels in the quantizer is given by

$$L_1 = 1 + \frac{2x_{max}}{\Delta} \quad \text{———} \quad (4)$$

Since the number of levels, L_1 is odd for midtread quantizer,

$$L = 2^n - 1 \quad \text{———} \quad (5)$$

By equating eq^n

$$2^n - 1 = 1 + \frac{2x_{max}}{\Delta}$$

$$2^n = 2 + \frac{2x_{max}}{\Delta}$$

$$2^n = 2\left[1 + \frac{x_{max}}{\Delta}\right]$$

$$\frac{2^n}{2} = 1 + \frac{x_{max}}{\Delta}$$

$$2^n\,2^{-1} = 1 + \frac{x_{max}}{\Delta}$$

$$2^{n-1} - 1 = \frac{x_{max}}{\Delta}$$

$$\left\langle \Delta = \frac{x_{max}}{2^{n-1} - 1} \right\rangle \quad \text{———} \quad (6)$$

If the probability of overload is $< 10^{-4}$, $x_{max} = 4\sigma_x$

$$\Delta = \frac{4\sigma_x}{2^{n-1}-1}$$

we know that.

$$(SNR)_0 = \frac{12\sigma_x^2}{\Delta^2}$$

$$(SNR)_0 = \frac{12\sigma_x^2}{\left(\frac{4\sigma_x}{2^{n-1}-1}\right)^2} = \frac{\overset{3}{\cancel{12}}\,\sigma_x^2}{\frac{\cancel{4}\,16\,\sigma_x^2}{(2^{n-1}-1)^2}}$$

$$\left\langle (SNR)_0 = \frac{3}{4}\,[2^{n-1}-1]^2 \right\rangle \text{———} \quad (7)$$

$$(SNR_0) = \frac{3}{4}\,[2^n \cdot 2^{-1} - 1]^2$$

$$= \frac{3}{4\times 2}\,[2^n-1)^2 = \frac{3}{16}\,[2^n-1)^2$$

For larger n i.e $n \geq 6$, we can neglect 1 i.e $2^n - 1 = 2^n$

$$(SNR)_0 = \frac{3}{16}\,2^{2n}$$

$$(SNR)_{dB} = 10\log_{10}(3/16) + 10\log_{10}(2^{2n})$$

$$= -7.2 + 20n\log_{10}(2)$$

$$= -7.2 + 6.02n$$

$$(SNR)_{dB} = 6.02n - 7.2.$$

A signal $x(t)$ is uniformly distributed in the range $\pm x_{max}$. Calculate signal to Noise ratio for pulse code modulation of this signal.

> we know that.

$$\left\langle SNR = \frac{\sigma_X^2}{\sigma_Q^2} \right\rangle \quad \text{——— (1)}$$

The PDF of $x(t)$ is

$$f_X(x) = \begin{cases} 1/2x_{max} & \text{for } -x_{max} \le x \le +x_{max} \\ 0 & \text{otherwise.} \end{cases}$$

* The mean square value of $x(t)$ is

$$\sigma_X^2 = \int_{-x_{max}}^{x_{max}} (x-\mu)^2 \, f_X(x) . dx \qquad \mu = 0$$

$$= \int_{-x_{max}}^{+x_{max}} (x-0)^2 \, \frac{1}{2x_{max}} \cdot dx$$

$$= \frac{1}{2x_{max}} \int_{-x_{max}}^{x_{max}} x^2 . dx = \frac{1}{2x_{max}} \left[\frac{x^3}{3} \right]_{-x_{max}}^{x_{max}} = \frac{1}{2x_{max} \times 3} \left[x_{max}^3 - (-x_{max})^3 \right]$$

$$\sigma_X^2 = \frac{1}{6x_{max}} \cdot 2x_{max}^3$$

$$\left\langle \sigma_X^2 = \frac{x_{max}^2}{3} \right\rangle \quad \text{——— (2)}$$

The PDF of Quantization error 'Q' is

$$f_Q(q) = \begin{cases} 1/\Delta & -\Delta/2 \le q \le \Delta/2 \\ 0 & \text{Otherwise} \end{cases}$$

The variance of quantization error is

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} (q-\mu)^2 \, f_Q(q) \, . dq \qquad \mu = 0$$

$$= \int_{-\Delta/2}^{\Delta/2} (q-0)^2 \cdot \frac{1}{\Delta} \cdot dq$$

$$= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 \cdot dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\cancel{2}\Delta^3}{\cancel{8}\,4}\right]$$

$$\left\langle \sigma_Q^2 = \frac{\Delta^2}{12} \right\rangle \text{———————} (3)$$

Substituting equation (2) and (3) in equation (1), we get.

$$SNR = \frac{\sigma_X^2}{\sigma_Q^2}$$

$$= \frac{x_{max}^2/3}{\Delta^2/12} = \frac{x_{max}^2}{\cancel{3}_1} \times \frac{\cancel{12}\,4}{\Delta^2} = \frac{4x_{max}^2}{\Delta^2}$$

$$\left\langle SNR = \frac{4x_{max}^2}{\Delta^2} \right\rangle \text{———————} (4)$$

we know that, $\Delta = \dfrac{2x_{max}}{M} \longrightarrow (5)$

Substituting equation (5) in equation (4).

$$SNR = \frac{4x_{max}^2}{\left(\frac{2x_{max}}{M}\right)^2} = \frac{4\cancel{x_{max}^2}}{\frac{4\cancel{x_{max}^2}}{M^2}} = M^2$$

$$\langle SNR = M^2 \rangle$$

we know that $M = 2^N$

$$(SNR) = [2^N]^2$$

$$\langle (SNR) = 2^{2N} \rangle$$

$$(SNR)_{dB} = 10 \log_{10} [2^{2N}]$$

$$= 2N \cdot 10 \log_{10} (2)$$

$$= 20N (0.30)$$

$$= N \times 20 \times 0.3$$

$$\langle (SNR)_{dB} = 6N \rangle$$

Bandwidth of PCM signal.

* The quantizer uses 'N' number of binary digits to represent each level

∴ The number of levels that can be represented by 'N' digits will be 'L'. i.e $L = 2^N$

Eg: If $N=3$, then total number of levels L

$$L = 2^3 = 8 \text{ levels}.$$

* The number of bits per second is called signalling rate of PCM & is denoted by 'r' i.e $r = Nf_s$ bits/sec

* Bandwidth needed for PCM transmission will be given by half of the signalling rate.

∴ Transmission bandwidth of PCM is

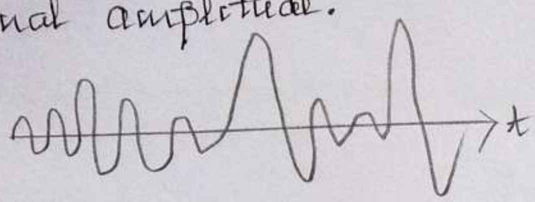
$$B_T \geq \frac{1}{2} r$$

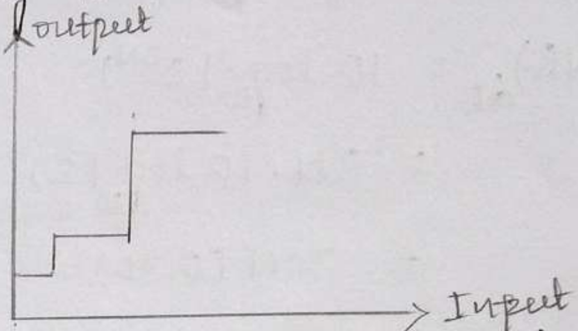
$$B_T \geq \frac{1}{2} Nf_s \qquad \langle B_T \geq NW \rangle$$

$$B_T \geq \frac{1}{2} \cdot N \cdot 2W$$

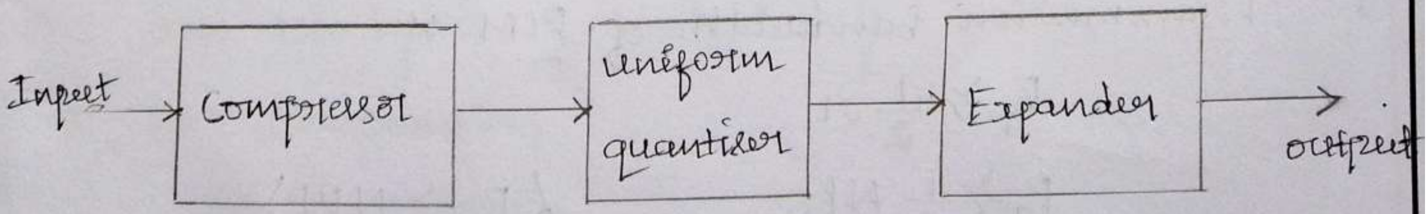
Non-uniform quantization or Robust quantization.

In Non-uniform quantization, the step size is not fixed. It varies according to certain law or as per signal amplitude.

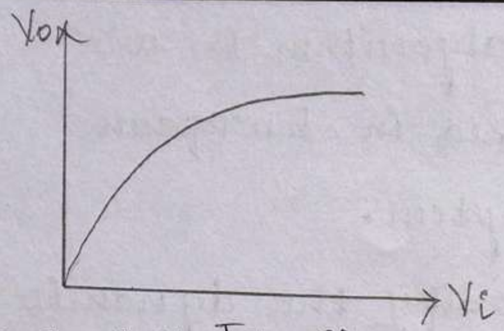




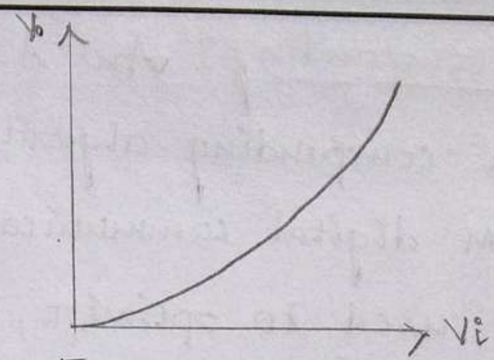
* The step size is small at low input signal levels. Hence quantization error is also small at these inputs. Therefore signal to quantization noise [SNR] power ratio is improved at low signal levels.
* The step size is higher at high input levels. Hence signal to noise [SNR] power ratio remains same throughout the dynamic range of quantizer.
* Maintaining constant signal to noise power ratio [SNR] throughout the signal range is called Robust quantization.
* To achieve constant SNR, the signal is passed through a combination of compressor-Expander circuit respectively at the transmitter & receiver. This process of compressing and expanding is called companding.



Model of Non-uniform quantizer



Compressor Transfer characteristics



Expander Transfer characteristics.

* To achieve robust quantisation, the signal is passed through a network which has an input-output characteristics as shown in above fig.
* The signal is changed such that small amplitude signals are boosted up without altering the maximum amplitude of the signal, Small amplitude signals range through more quantisation levels.
* Any signal when passed through such a network gets compressed leading to signal distortion.
* To remove this distortion, the signal is passed through an Inverse network at the receiver called as Expander.
* The complete process of compressing and expanding the signal is referred to as companding

There are two types of companding

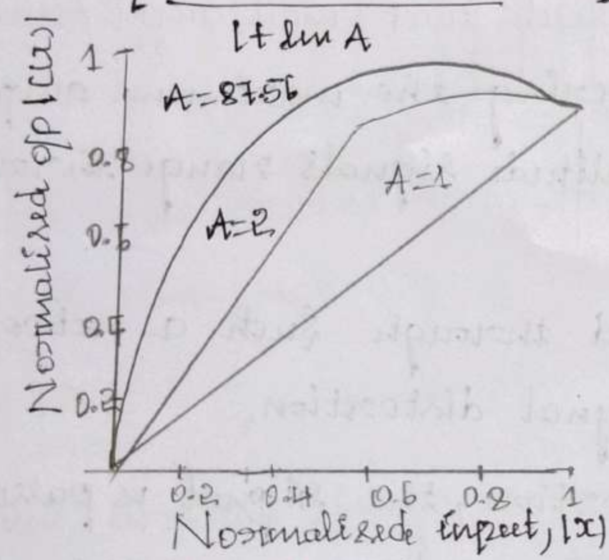
1) A-law companding
2) μ-law companding.

A-law companding: An A-law algorithm is a standard companding algorithm, used in European 8-bit PCM digital communication system.

* It is used to optimize, i.e modify the dynamic range of an analog signal for digitizing.

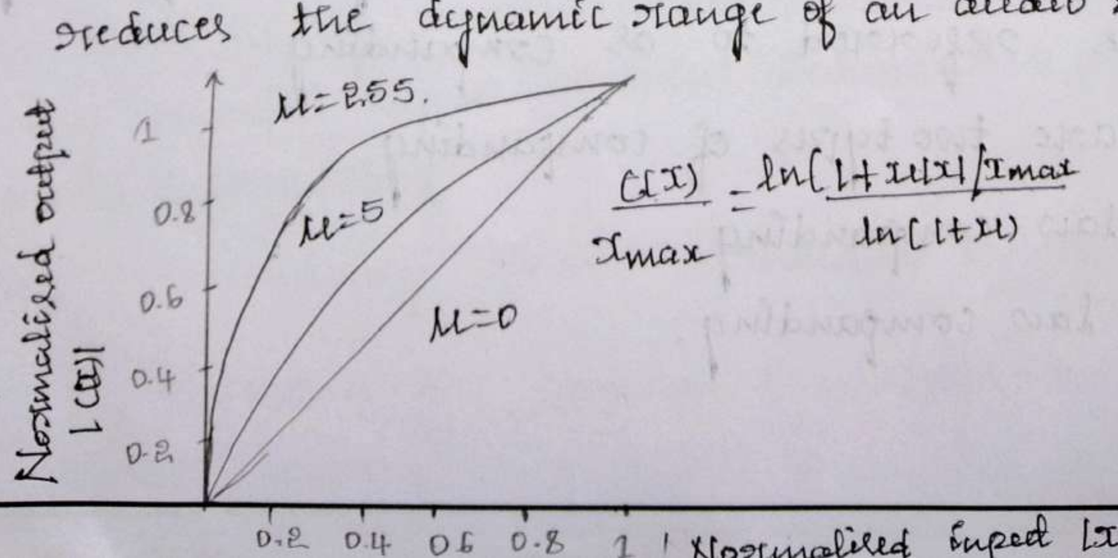
A law is defined by following equation.

$$\frac{C(|x|)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\ln A} & 0 \le \dfrac{|x|}{x_{max}} \le 1/A \\ \dfrac{1+\ln[A|x|/x_{max}]}{1+\ln A} & \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$



μ-law companding: The μ-law algorithm is a companding algorithm, primarily used in 8-bit PCM digital telecommunication system in North America and Japan.

It reduces the dynamic range of an audio signal.



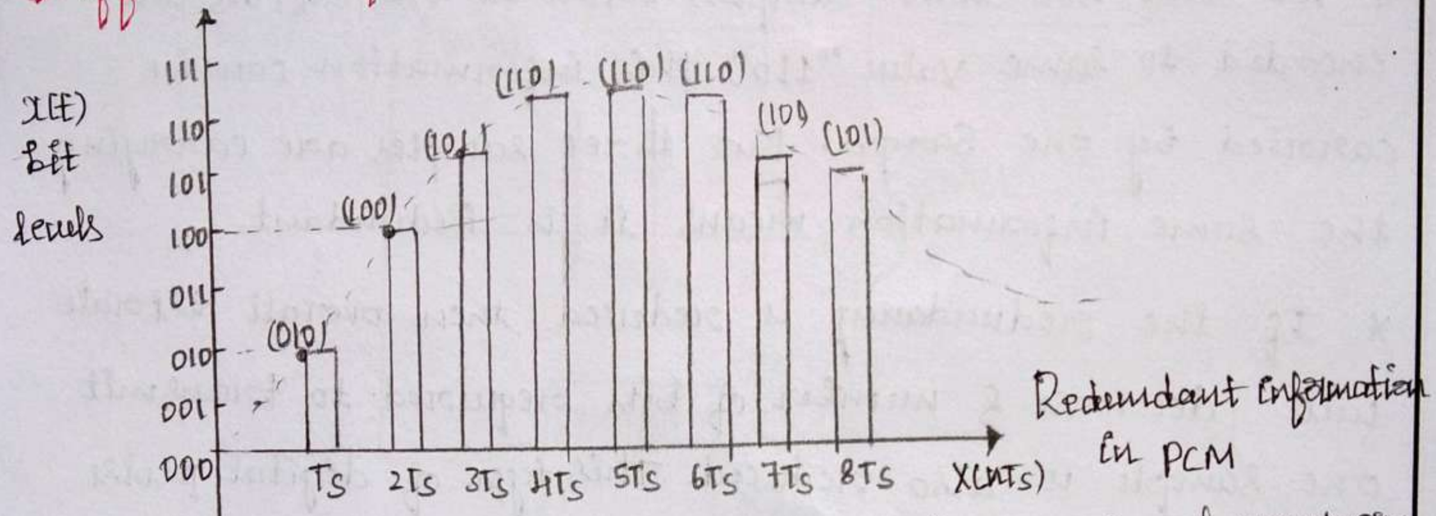
$$\frac{C(x)}{x_{max}} = \frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)} \quad 0 \le \frac{|x|}{x_{max}} \le 1$$

Advantages of Non-uniform quantization.

1) Reduced quantization noise

2) High average SNR.

Differential pulse code modulation.



* In pulse code modulation system each sample of waveform is encoded independently of other samples.

* The samples of signals are highly correlated, this is because
  i) Signal does not change fast.
  ii) The samples above the Nyquist rate i.e $f_s > 2W$

thus, the signal does not change rapidly from one sample to the next sample.

* when these highly correlated samples are encoded, the resulting encoded signal contains Redundant information. [duplicated information]

* The redundancy can be eliminated by DPCM.

* The fig(1) shows the continous time signal x(t) sampled used flattop sampling. This signal is sampled at the

instants $T_s, 2T_s, 3T_s, \ldots\ldots nT_s$. The Sampling frequency is selected higher than the Nyquist rate & encoded using 3-bit encoder. The samples quantised to nearest level is shown in fig(1)

* We can see that samples taken at $4T_s, 5T_s, 6T_s$ are encoded to same value "110". this information can be carried by one sample. But three samples are carrying the same information means it is Redundant.

* If the redundancy is reduced, then overall bit rate will decrease & number of bits required to transmit one sample is also reduced. This type of digital pulse code modulation scheme is known as DPCM[Differential pulse code modulation].

Differential pulse code modulation.

* When an analog signal is sampled at a rate slightly higher than the Nyquist rate, resulting high correlated samples. i.e the signal does not change rapidly from one sample to next.

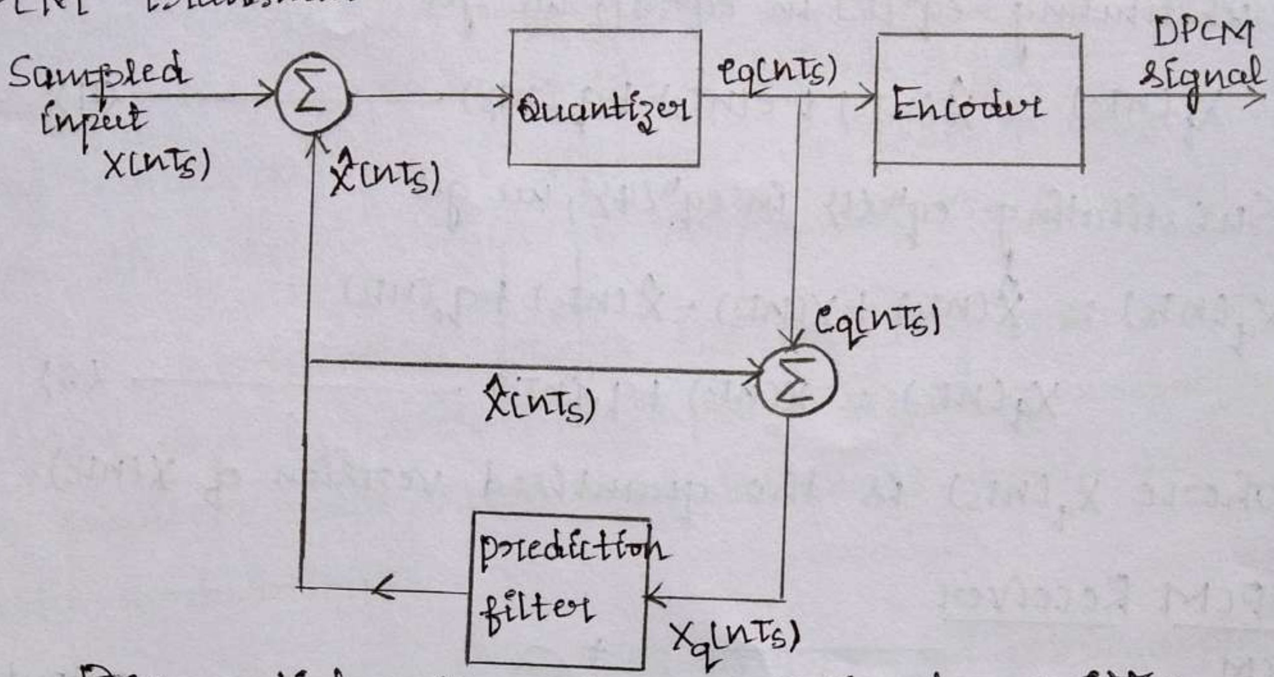
* If these highly correlated samples are encoded, the resulting encoded signal contains redundant information.

* If we remove this redundancy before encoding, efficiency of the coded signal can be increased.

* DPCM works on the principle of prediction. The value of the present sample is predicted from the

past values. The prediction may not exact but it is very close to the actual sample value.

DPCM transmitter



Differential pulse code modulation transmitter.

* $x(nT_s)$ represents the sampled version of the analog signal $x(t)$ [shown in fig (1)]

* The output of the comparator is the difference between the sampled input and prediction of its, $\hat{x}(nT_s)$

i.e $\quad e(nT_s) = x(nT_s) - \hat{x}(nT_s)$

where $\hat{x}(nT_s)$ is the prediction of $x(nT_s)$
$e(nT_s)$ is the prediction error.

* The prediction error $e(nT_s)$ is the quantized to produce $e_q(nT_s)$

* In the quantizer, the noise $q_e(nT_s)$ gets added

∴ The o/p of the quantizer can be written as

$$e_q(nT_s) = e(nT_s) + q_e(nT_s)$$

The Input to the prediction filter is written as

$$X_q(nT_s) = \hat{X}(nT_s) + e_q(nT_s) \quad \text{——— } \langle 3 \rangle$$

Substituting eq^n[2] in eq^n[3], we get

$$X_q(nT_s) = \hat{X}(nT_s) + e(nT_s) + q_e(nT_s) \quad \text{——— } \langle 4 \rangle$$

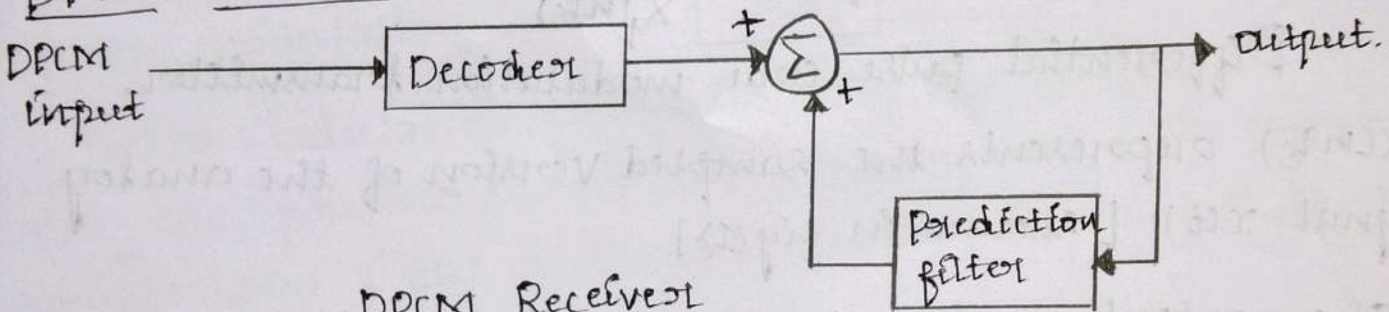
Substituting eq^n⟨1⟩ in eq^n⟨4⟩, we get

$$X_q(nT_s) = \hat{X}(nT_s) + X(nT_s) - \hat{X}(nT_s) + q_e(nT_s)$$

$$X_q(nT_s) = X(nT_s) + q_e(nT_s) \quad \text{——— } \langle 5 \rangle$$

where $X_q(nT_s)$ is the quantised version of $X(nT_s)$

DPCM Receiver



DPCM Receiver

* The decoder reconstructs the quantized error signal from incoming binary signal.
* The prediction filter output & quantised error signals are summed up to give the quantized version of the original signal

$$\langle e_q(nT_s) + \hat{X}(nT_s) = X_q(nT_s) \rangle$$

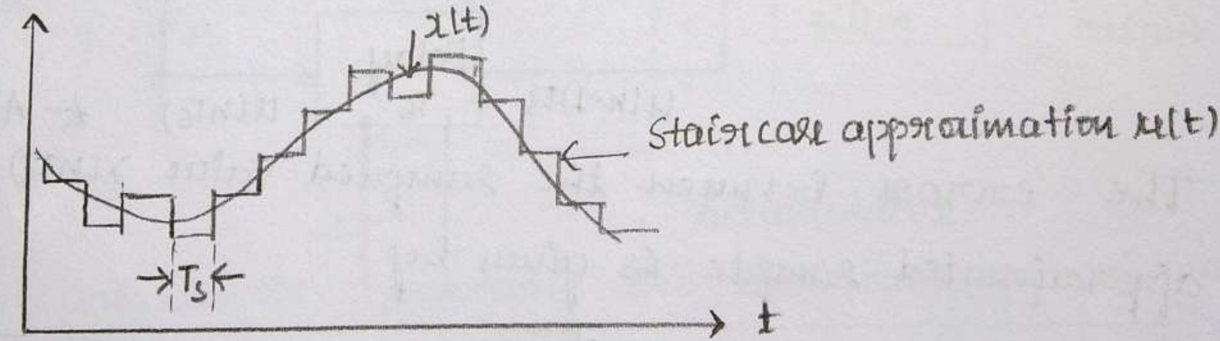
* Thus, the signal at the receiver differs from actual signal by quantization error $q_e(nT_s)$, which is introduced permanently in the reconstructed signal.

Delta modulation.

The Sampling rate of a signal should be higher than the Nyquist rate, to achieve better sampling. If the sampling interval in Differential PCM is reduced considerably, the sample to sample amplitude difference is very small, as if the difference is 1-bit quantization, then the stepsize will be very small. i-e Δ [delta]

The modulation, where the sampling rate is much higher and in which the stepsize after quantization is smaller value, such modulation is termed as Delta modulation.

Delta modulation transmits only one bit per sample. i.e the present sample value is compared with previous sample value & the indication, whether the amplitude is increased or decreased is sent.

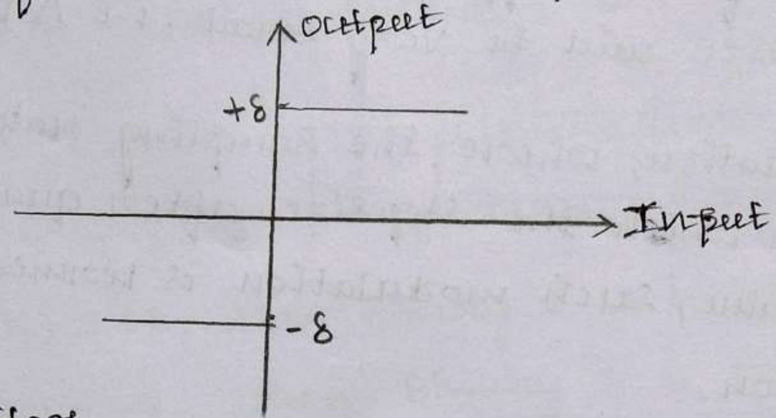


Binary sequence at modulator output 0 0 1 0 1 1 1 1 0 1 0 0 0 0

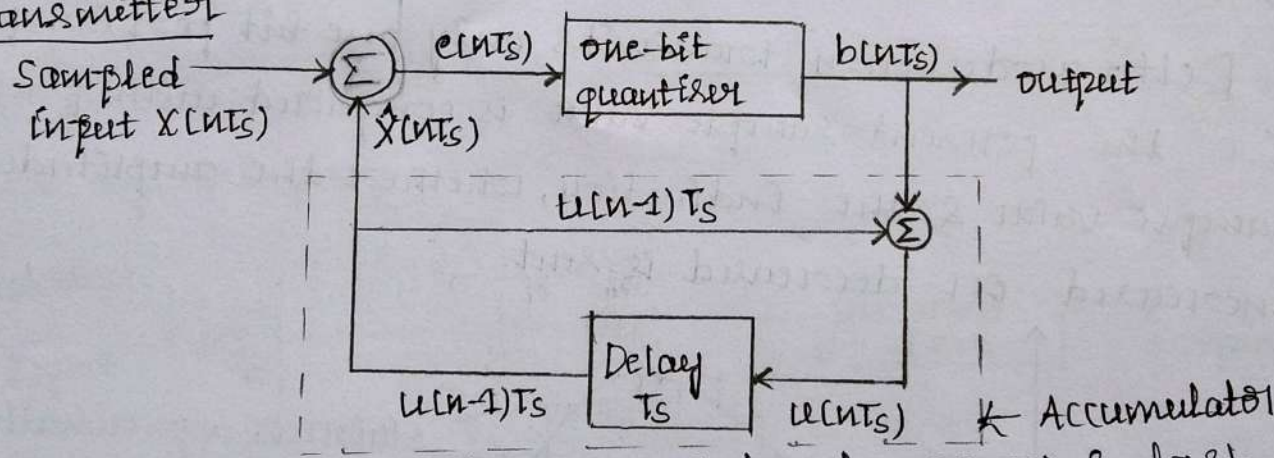
1) The Input signal x(t) is approximated to step signal by the delta modulator.

2) the difference between Input signal x(t) and staircase approximated signal is quantized into only two levels i-e $+\delta$ or $-\delta$.

* If the difference is +ve, then approximated signal is increased by one step i.e $+\delta$ & bit 1 is transmitted.
* If the difference is −ve, then approximated signal is reduced by one step i.e $-\delta$ & bit 0 is transmitted.
* thus for each sample only one-bit is transmitted.



Transmitter



The error between the sampled value $x(nT_s)$ & last approximated sample is given by

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{——} \langle 1 \rangle$$

Let $u(nT_s)$ be the present sample approximation of staircase output.

From the figure : $\hat{x}(nT_s) = u(n-1)T_s$

$$\hat{x}(nT_s) = u(nT_s - T_s) \quad \text{——} \langle 2 \rangle$$

Substituting eq$^n\langle 2 \rangle$ in eq$^n\langle 1 \rangle$, we get

$$e(nT_s) = x(nT_s) - u(nT_s - T_s) \quad \text{——} \langle 3 \rangle$$

* The binary quantity $b(nT_s)$ is the algebraic sign of the error $e(nT_s)$, except for the scaling factor δ

i.e $b(nT_s) = \delta\, \mathrm{Sgn}[e(nT_s)]$ ——— <4>

$b(nT_s)$ depends on the sign of error $e(nT_s)$, the sign of step size 'δ' will be decided.

i.e $b(nT_s) = +\delta$, if $x(nT_s) \geq \hat{x}(nT_s)$

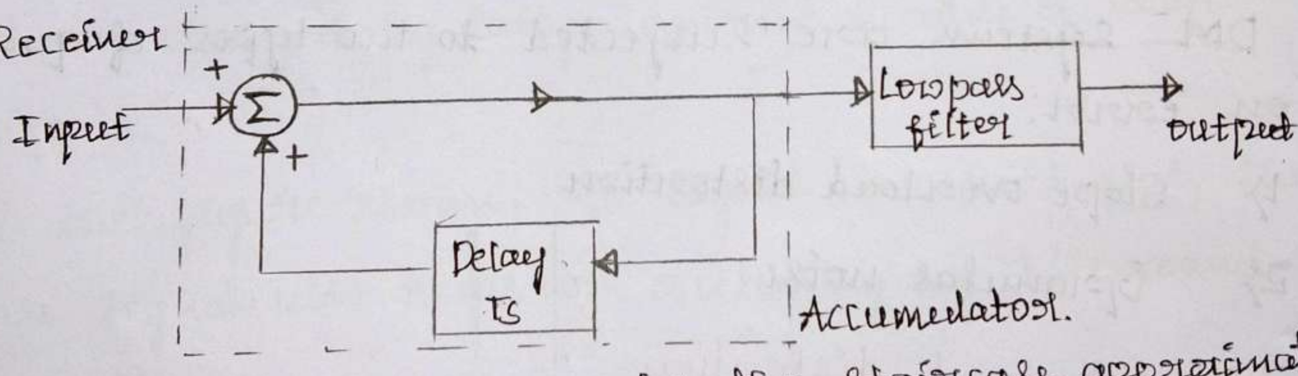
$b(nT_s) = -\delta$, if $x(nT_s) \leq \hat{x}(nT_s)$

* If $b(nT_s) = +\delta$, then binary '1' is transmitted.

If $b(nT_s) = -\delta$, then binary '0' is transmitted.

* The previous sample approximation $u[nT_s - T_s]$ is restored by delaying one sample period 'T_s'.

DM Receiver



* The accumulator generates the staircase approximated signal output and is delayed by one sampling period 'T_s'. It is then added to the input signal.

* If input is binary '1' then it adds $+\delta$ step to the previous output.

* If input is binary '0', then one step 'δ' is subtracted from the delayed signal.

The Low pass filter is used to remove step variation & to get smooth reconstructed message signal x(t).

Advantages of Delta modulation[DM]

The DM has following advantages over PCM

1> DM transmits only one bit for one sample. Thus the signalling rate & transmission channel bandwidth is quite small for DM

2> Simplicity of design for both the transmitter and the receiver

3> A one-bit code word for the output, which eliminates the need for word framing.

Drawbacks of DM:

DM systems are subjected to two types of quantization error:

1> Slope overload distortion

2> Granular noise.

Slope overload distortion

Slope overload distortion arises because of large dynamic range of the input signal.

As shown in fig, the rate of rise of input signal x(t) is so high that the staircase signal cannot approximate it, the step size δ becomes too small for staircase signal x(t) to follow the steep segment of x(t).

Thus large error between the staircase approximated signal & the original I/p signal $x(t)$. This error is called slope overload distortion.

* To reduce this error, the step size should be increased when slope of the signal $x(t)$ is high.

i.e Slope of the staircase $u(t)$ ≥ Slope of the message signal

$$\frac{\delta}{T_s} \geq \max \left| \frac{d[x(t)]}{dt} \right|$$

Granular noise

This noise occurs when the step size is too large compared to small variations in the I/p signal. i.e for very small variations in the I/p signal, the staircase signal is changed by large amount because of large step size 'δ'.

* In the figure shown, I/p signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\delta$ around the signal.

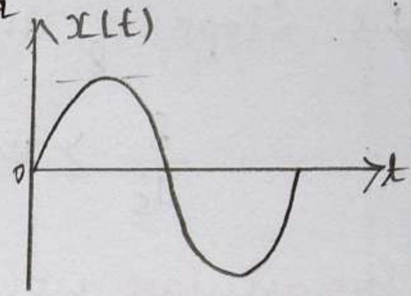
* The error between the I/p and approximated signal is called granular noise. The solution of this problem is to make stepsize small.

Consider a sine wave of frequency f_m & amplitude A_m applied to a delta modulator of step size δ.

ST Slope overload distortion will occur if

$$A_m > \frac{\delta}{2\pi f_m T_s}$$ where T_s is the sampling period

* Let the message signal be represented as $x(t) = A_m \sin(2\pi f_m t)$



* consider uniform DM with step size δ & sampling interval T_s.

$\therefore$ The slope of DM $= \frac{\text{Step size}}{\text{Sampling period}} = \frac{\delta}{T_s}$

* The slope overload distortion will takes place if slope of sine wave is greater than slope of demodulator

i.e $\quad \max\left|\frac{d}{dt}(x(t))\right| > \frac{\delta}{T_s}$

$$\max\left|\frac{d}{dt} A_m \sin(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$\max\left| A_m \cos 2\pi f_m t \cdot 2\pi f_m\right| > \frac{\delta}{T_s}$$

Slope will be maximum when $\cos 2\pi f_m t = 1$
i.e at $t = 0$.

$$A_m (1) \cdot 2\pi f_m > \frac{\delta}{T_s}$$

$$\boxed{A_m > \frac{\delta}{2\pi f_m T_s}}$$

Derive an expression for signal to [noise] quantization noise power ratio for delta modulation. Assume that no slope overload distortion exists.

> The signal power $x(t) = A_0 \cos(2\pi f_0 t)$

The maximum slope of the signal $x(t)$ is given by

$$\left| \frac{d(x(t))}{dt} \right|_{max} = \left| \frac{d}{dt} A_0 \cos(2\pi f_0 t) \right|_{max}$$

$$= \left| -A_0 \sin 2\pi f_0 t \cdot 2\pi f_0 \right|_{max}$$

$\sin 2\pi f_0 t = 1$ at $t = 90^\circ$

$$\boxed{\left| \frac{d}{dt} (x(t)) \right|_{max} = A_0 2\pi f_0} \quad \text{——} \langle 1 \rangle$$

* The condition for no slope overload distortion is

$$\boxed{\frac{\delta}{T_s} \geq \left| \frac{d[x(t)]}{dt} \right|} \quad \text{——} \langle 2 \rangle$$

Substituting eq ① in eq ②, we get.

$$\frac{\delta}{T_s} \geq A_0 2\pi f_0$$

$$A_0 2\pi f_0 \leq \frac{\delta}{T_s}$$

$$A_0 = \frac{\delta}{2\pi f_0 T_s}$$

$$\left\langle A_0 = \frac{\delta f_s}{2\pi f_0} \right\rangle \quad \text{——} \langle 3 \rangle$$

* The maximum average power of the signal $x(t)$ is given by

$$P_{max} = \frac{V^2}{R}$$

$V \rightarrow$ Rms value of the signal $\therefore V = \frac{a_0}{\sqrt{2}}$

* Normalised signal power is obtained by taking $R = 1$

$$P_{max} = \left(\frac{A_0}{\sqrt{2}}\right)^2 / 1$$

$$P_{max} = \frac{A_0^2}{2} \longrightarrow \langle 4 \rangle$$

Substituting eq ③ in eq ④, we get.

$$P_{max} = \frac{\delta^2 f_s^2}{4\pi^2 f_0^2} \times \frac{1}{2}$$

$$\left\langle P_{max} = \frac{\delta^2 f_s^2}{8\pi^2 f_0^2} \right\rangle \longrightarrow \langle 5 \rangle$$

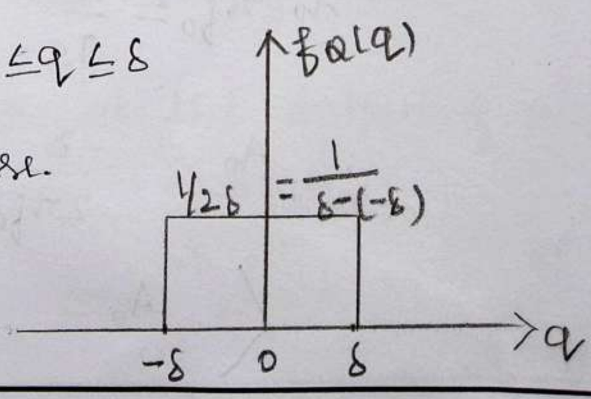
P_{max} is the signal power in delta modulation

Noise power

When there is no slope overload, the maximum quantization error is $\pm\delta$. Assuming that the quantization error is uniformly distributed.

* Consider the PDF of the quantization error i.e

$$f_Q(q) = \begin{cases} \frac{1}{2\delta} & \text{for } -\delta \le q \le \delta \\ 0 & \text{otherwise.} \end{cases}$$



The variance of quantization error is

$$\sigma_Q^2 = \int_{-\delta}^{\delta} q^2 \cdot f_Q(q) \cdot dq$$

$$= \int_{-\delta}^{\delta} q^2 \cdot \frac{1}{2\delta} \cdot dq = \frac{1}{2\delta} \int_{-q}^{q} q^2 \cdot dq$$

$$= \frac{1}{2\delta}\left[\frac{q^3}{3}\right]_{-\delta}^{\delta} = \frac{1}{2\delta}\left[\frac{\delta^3}{3} - \frac{(-\delta)^3}{3}\right]$$

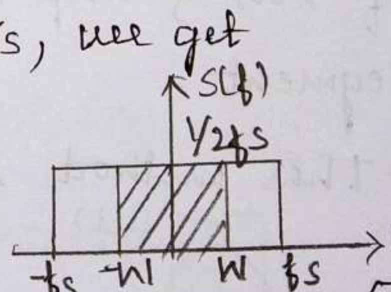
$$= \frac{1}{2\delta}\left[\frac{\delta^3}{3} + \frac{\delta^3}{3}\right]$$

$$= \frac{1}{\cancel{2}\delta}\left[\frac{\cancel{2}\delta^3}{3}\right]$$

$$\left\langle \sigma_Q^2 = \delta^2/3 \right\rangle \quad \text{—— } \langle 6 \rangle$$

* The DM receiver contains a LPF whose cut-off frequency $f_C = W$ Hz.
* Assuming that the average power of the quantization error is uniformly distributed over a frequency interval extending from $-1/T_s$ to $+1/T_s$, we get

Average output noise power 'N_0'



$$N_0 = \left(\frac{f_C}{f_s}\right)\frac{\delta^2}{3} = W T_s \frac{\delta^2}{3} \quad \text{—— } \langle 7 \rangle$$

* Signal to noise power ratio at the output of DM receiver is

$$\left\langle (SNR)_0 = \frac{P_{max}}{N_0} \right\rangle$$

$$(SNR_o) = \frac{\delta^2 f_s^3 / 8\pi^2 f_0^2}{WT_S\, \delta^2/3} = \frac{\delta^2 f_s^2}{8\pi^2 f_0^2} \times \frac{3}{WT_S\, \delta^2}$$

$$(SNR_o) = \frac{3}{8\pi^2 f_0^2 T_S^2 \cdot WT_S}$$

$T_S^2 = 1/f_S^2$

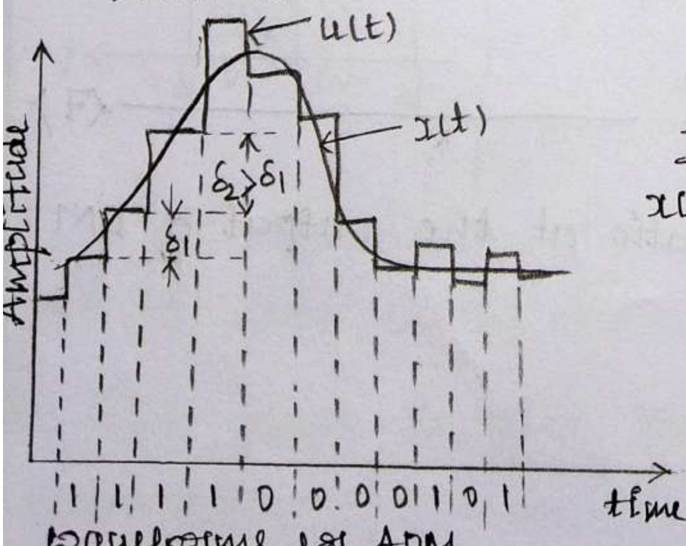
$$(SNR_o) = \frac{3}{8\pi^2 f_0^2 W T_S^3} \qquad \text{—————} \quad \langle 8 \rangle$$

Eq^n (8) shows that, under the assumption of no-slope overload distortion, the minimum output signal to noise ratio of a delta modulator is proportional to the sampling rate cubed.

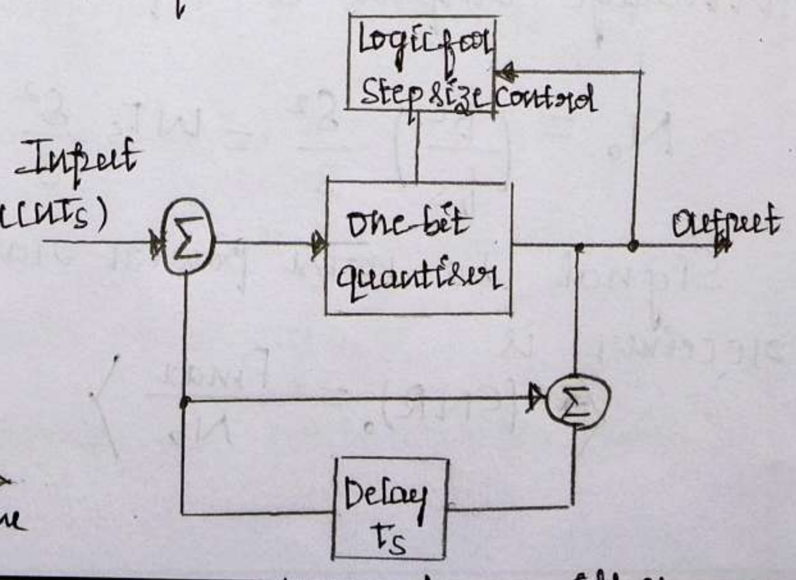
Adaptive delta modulation.

* To overcome slope overload distortion & granular noise, step size δ is made adaptive to variations in the input signal $x(t)$.
* The step size is increased for the steep segment of $x(t)$ & step size is decreased for slowly varying segment.

This method is called Adaptive delta modulation.



waveforms for ADM



ADM transmitter.

* In practical adaptive DM, the step size $\delta(nT_s)$ is constrained to lie between maximum and minimum values

i.e $\delta_{min} \leq \delta(nT_s) \leq \delta_{max}$

* this upper limit δ_{max}, controls the amount of slope overload [condition] distortion.

* the lower limit δ_{min}, controls the amount of granular noise

* Inside these limits the adaptive algorithm rule of $\delta(nT_s)$ expressed in general form

$$\boxed{\delta(nT_s) = g(nT_s)\,\delta(nT_s - T_s)}$$

Where the time varying function $g(nT_s)$ depends on the present & previous binary o/p of the delta modulator.

* The adaptive algorithm is initiated with a starting step $\boxed{\delta_{start} = \delta_{min}}$

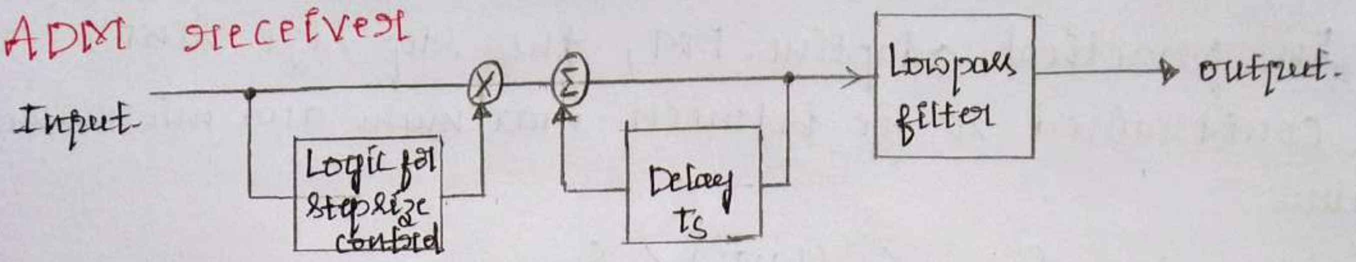
* The function $g(nT_s)$ is given by

$$g(nT_s) = \begin{cases} K & \text{if } b(nT_s) = b(nT_s - T_s) \\ K^{-1} & \text{if } b(nT_s) \neq b(nT_s - T_s) \end{cases}$$

* This adaptive algorith is called a constant factor ADM with one-bit memory

* The ADM is also known as continous variable slope DM

ADM receiver



* In the receiver, the 1st part generates step size from each incoming bit which is variable in size. The previous input & present input decides the step size.
* The LPF then smoothens out the staircase waveform to reconstruct the smooth signal.

Advantages of ADM

* The signal to noise ratio is better than ordinary delta modulation because of the reduction in the slope overload distortion & granular noise.
* Utilization of bandwidth is better than Delta modulation